representation is always simpler and more explicit than an integral representation if only a few terms are sufficient to represent the boundary condition with a good approximation.

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Effect of Uncooled Leading Edge on Cooled-Wall Hypersonic Flat-Plate Boundary-Layer Transition

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Nomenclature

b = leading-edge bluntness diam, in.
M = Mach number
Pr = Prandtl number
Re = Reynolds number
T = temperature, °R
u = velocity parallel to model, fps
x = distance from leading-edge tip, parallel to model, in.
x' = distance from leading-edge-model joint, parallel to model, in.
y = distance from plate surface, normal to surface, in.
γ = ratio of air specific heats
δ = boundary-layer thickness (point at which $\rho u/\rho_\infty u_\infty = 0.99$), in.
ρ = density of air, lbf sec²/ft⁴

Subscripts

aw = adiabatic wall condition, assuming recovery factor of 0.85
j = value at joint between leading edge and model plate section
LE = condition of leading-edge section
o = wind-tunnel stagnation condition
t = value at transition point (defined in text)
w = condition of model plate section
∞ = wind-tunnel freestream conditions

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Hypersonic, flat-plate boundary-layer transition experiments were performed in the Arnold Engineering Development Center's von Kármán Facility 12-in. × 12-in. hypersonic wind tunnel at Mach numbers 6, 7, and 8. The flat-plate models were designed to allow investigation not only of wall cooling effects but also the effect of varying small leading-edge bluntness. The models were made up of a common flat-plate section with internal coolant flow passages and interchangeable, uncooled leading-edge sections, each with a different hemicylindrical bluntness radius and of 2-in. length.

Upon completion of the tests, examination of the data showed very little effect of wall cooling on the transition Reynolds number (defined as the product of unit Reynolds number and distance along the plate to the point of maximum pressure indicated by a Pitot tube drawn along the model surface) compared to that observed by other experimenters. As shown in Fig. 1 for a Mach number of 8, the data of Richards and Stollery[1] show a change in transition Reynolds number for a change in T_w/T_{aw} from 1 to 0.9 which is accomplished in the present experiments only by cooling to about $T_w/T_{aw} = 0.6$. Granted there are many factors influencing transition Reynolds number, some of which are active in causing the difference in absolute level of transition Reynolds number between the present test and those of Ref. 1 (see Ref. 2 for the influence of radiated aerodynamic noise); still, the question arose concerning a possible role the uncooled leading edge might have in reducing the influence of downstream cooling, particularly since the transition location method used in the present tests required model exposure times on the order of five minutes.

Since the leading-edge temperature was not measured, an estimate of its value at the different test Mach numbers was made by assuming it equal to the temperature the plate section attained when run without cooling. This assumption leads to estimates of leading-edge temperature ratio, T_{LE}/T_{aw}, of 0.9, 0.8, and 0.7 for $M_\infty = 6$, 7, and 8, respectively, for all tests regardless of the temperature subsequently reached by the cooled plate section.

Digital computer calculations were later made of the characteristics of the laminar boundary layer for a Mach number of 6, unit Reynolds number of 1.1×10^6/in., assuming a perfect gas with ratio of specific heats 1.4 and Prandtl number 0.71. These calculations were made by C. H. Lewis[3] of the von Kármán Facility, using the method of Ref. 4, for the following conditions: leading edge and plate at the same temperature for uniform wall temperatures of 0.8 and 0.2 times the freestream stagnation temperature and with the leading edge at 0.8 and the remainder of the plate at 0.2 times the freestream stagnation temperature. The resulting velocity-density product profiles are compared in Fig. 2 for a station

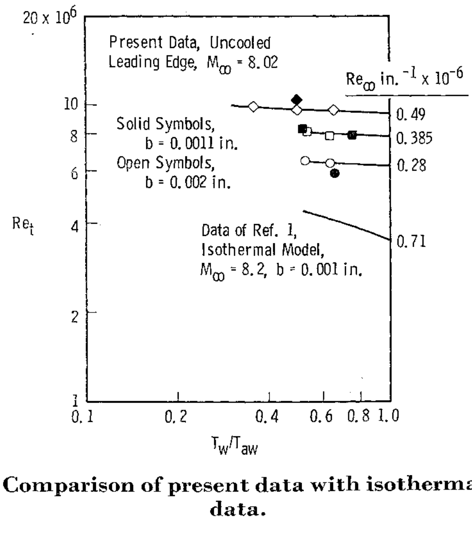

Fig. 1 Comparison of present data with isothermal model data.

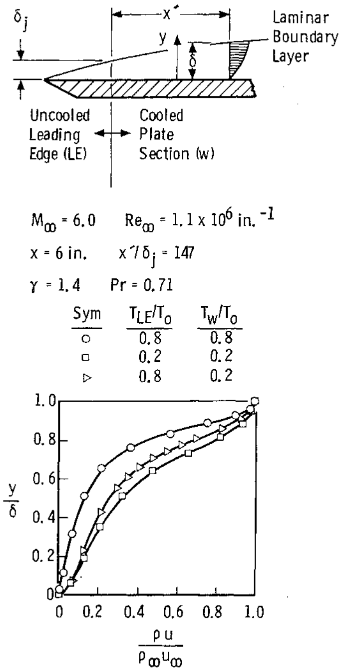

Fig. 2 Theoretical velocity-density product profiles for various model surface temperature distributions.

corresponding to the experimental beginning of transition and show that at $y/\delta = 0.9$, in the region of the critical layer or earliest disturbance-amplifying region, the boundary-layer profile has not adjusted completely to the new wall temperature condition. The percentage of adjustment is related in Fig. 3 to the parameter x'/δ_j, or distance from the leading-edge-to-plate joint divided by the boundary-layer thickness at the joint from Ref. 3. There it is seen that the laminar boundary layer approaching transition has, insofar as the $y/\delta \approx 0.9$ region is concerned, experienced conditions corresponding to considerably higher wall temperatures than existed at the wall. The boundary layer has developed over the two-in. leading edge at hot-wall conditions and then has proceeded to the beginning of transition with an average ($0 \leq x'/\delta_j \leq 147$) outer-region accommodation to the new wall temperature of only 55% and a local value at the beginning of transition ($x'/\delta_j = 147$) of about 67%. A calculation of the reduction of uncooled leading-edge length required for 90% critical layer accommodation to the cold wall conditions, with distance to transition remaining fixed at $x = 6$ in., gives the

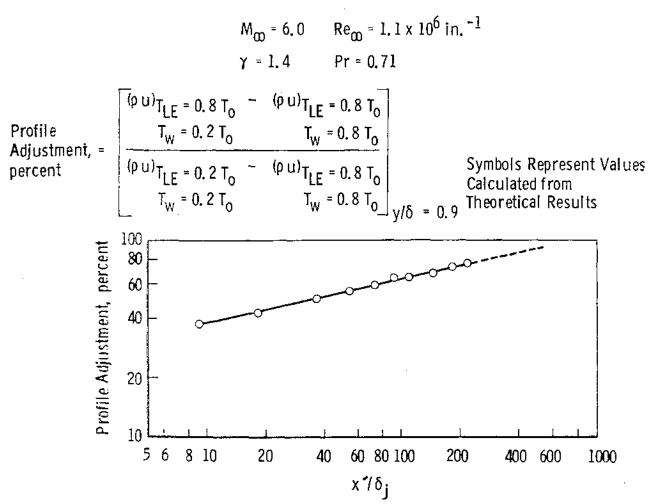

Fig. 3 Theoretical percentage-adjustment of laminar boundary layer to step change in wall temperature.

result that the uncooled length must be reduced from two in. to 0.35 in.

In conclusion, it appears that for boundary-layer transition experiments examining the effects of wall cooling, a careful consideration of possible hot leading-edge effects is indicated.

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Transient Stresses at a Clamped Support of an Orthotropic, Circular, Cylindrical Shell

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Introduction

Many composite materials have improved the efficiency of structural elements for aerospace vehicles. Since unlimited types of composite materials are possible, it is imperative to provide analyses which can guide the selection of efficient materials. In this analysis, the transient bending and shear stresses at the clamped support of a semi-infinite, orthotropic, circular, cylindrical shell produced by a uniform, radial impulse are calculated from the shell bending theory and a Timoshenko-type shell theory. These results provide design criteria for the shell requirements in the vicinity of stiff ring supports.

The title problem was recently investigated for an isotropic shell.[1] This Note generalizes the work in Ref. 1 by considering an orthotropic shell.

Shell Equations of Motion

Timoshenko-type equations which govern the axially symmetric motion of orthotropic, circular, cylindrical shells are presented in Ref. 2. The axial normal stress is taken as zero, and the shell equations reduce to

$$\frac{\kappa^2(1-\nu_{x\theta}\nu_{\theta x})G}{E_\theta}\left[\frac{\partial^2 W}{\partial\eta^2}-\frac{\partial\psi}{\partial\eta}\right]-(1-\nu_{x\theta}\nu_{\theta x})W = \frac{\partial^2 W}{\partial\tau^2}-\frac{c(1-\nu_{x\theta}\nu_{\theta x})}{hE_\theta}I\delta(\tau) \quad (1a)$$

$$\frac{h^2}{12a^2}\frac{\partial^2\psi}{\partial\eta^2}+\frac{\kappa^2 G(1-\nu_{x\theta}\nu_{\theta x})}{E_x}\left[\frac{\partial W}{\partial\eta}-\psi\right]=\frac{h^2 E_\theta}{12a^2E_x}\frac{\partial^2\psi}{\partial\tau^2} \quad (1b)$$

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